

ON THE ISOMORPHISM PROBLEM FOR GENERALIZED BAUMSLAG–SOLITAR GROUPS

F.A. Dudkin

Sobolev Institute of Mathematics, Siberian Branch of the Russian Academy of Sciences
4 Acad. Koptug avenue, 630090, Novosibirsk, Russia
Novosibirsk State University, Ministry of Education and Science of the Russian Federation
2 Pirogova Str., 630090, Novosibirsk, Russia
DudkinF@ngs.ru

Call a finitely generated group G a *generalized Baumslag-Solitar group* or a *GBS* group if G can act on a tree so that the stabilizers of vertices and edges are infinite cyclic groups. By the Bass-Serre theorem, G is representable as $\pi_1(\mathbb{A})$, the fundamental group of a graph of groups \mathbb{A} (see [1]).

Given a *GBS* group G , we can present the corresponding graph of groups \mathbb{A} by a labeled graph (A, λ) , where A is a finite connected graph and $\lambda: E(A) \rightarrow \mathbb{Z} \setminus \{0\}$ labels the edges of A . The label λ_e of an edge e with the source vertex v defines an embedding $\alpha_e: e \rightarrow v^{\lambda_e}$ of the cyclic edge group $\langle e \rangle$ into the cyclic vertex group $\langle v \rangle$. Using the notion of expansion for labeled graphs, we can easily see that every *GBS* group can be presented by infinitely many labeled graphs.

Recently *GBS* groups have been quite actively studied [2–4]. In particular, the isomorphism problem for *GBS* groups has been discussed: to determine algorithmically when two given labeled graphs define isomorphic *GBS* groups. Despite that, the isomorphism problem is solved only in several special cases [5–7], the general solution is not established.

If two labeled graphs \mathbb{A} and \mathbb{B} define isomorphic *GBS* groups $\pi_1(\mathbb{A}) \cong \pi_1(\mathbb{B})$ and $\pi_1(\mathbb{A})$ is not isomorphic to \mathbb{Z} , \mathbb{Z}^2 or Klein bottle group then there exists a finite sequence of *expansion* and *collapse* (see fig.1) moves connecting \mathbb{A} and \mathbb{B} [8]. A labeled graph is called *reduced* if it admits no collapse move (equivalently, the labeled graph contains no edges with distinct endpoints and labels ± 1).

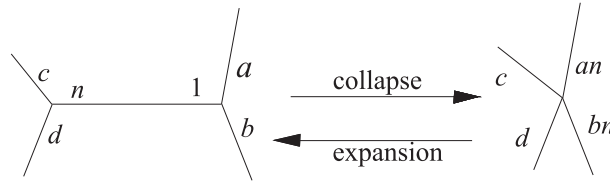


Fig. 1: Expansion and collapse moves.

Given a labeled graph \mathbb{A} (a *GBS* group G), denote the set of reduced labeled graphs with the fundamental group isomorphic to $\pi_1(\mathbb{A})$ (resp. G) by $R(\mathbb{A})$ (resp. $R(G)$).

Three types of transformations of labeled graphs plays an important role in studying *GBS* groups: slide (see fig. 2), induction, \mathcal{A}^\pm -moves.

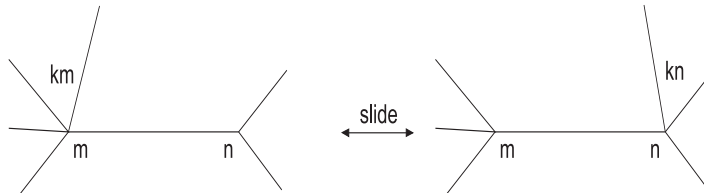


Fig. 2: Slide.

Theorem (Clay M., Forester M. [2]). *Given GBS group G and $\mathbb{A}, \mathbb{B} \in R(G)$, then \mathbb{A} and \mathbb{B} related by a finite sequence of slides, inductions and $\mathcal{A}^{\pm 1}$ -moves, with all intermediate labeled graphs reduced.*

An edge e of a labeled graph \mathbb{A} is called *mobile* (see [6]), if there exists $t \in \pi_1(\mathbb{A})$ such that $G_e^t \subset G_e$. There G_e is an edge cyclic group, corresponding to the edge e . In [6] it is proved that there is an algorithm to decide if given edge e mobile or not.

The main result of this paper is another piece of the isomorphism problem for GBS groups:

Theorem. *Given labeled graphs \mathbb{A} and \mathbb{B} . Suppose that \mathbb{A} has no more than one mobile edge. Then there is an algorithm to decide if groups $\pi_1(\mathbb{A})$ and $\pi_1(\mathbb{B})$ are isomorphic.*

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